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ALGEBRAIC RIEMANN MANIFOLDS

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Introduction

In the present paper, we are concerned with the problem to know whether two algebraic Riemann manifolds are isometric or not, where we mean Riemann manifolds of class C^{Ω} (real algebraic smoothness or Nash category) simply by algebraic Riemann manifolds.

For this purpose we introduce notions of minimal differential polynomials and singular base points, both of which are isometry invariants. With the aid of these invariants, we give the following isometry theorem for algebraic Riemann manifolds:

MAIN THEOREM. *Let M, $\overline{M}$ be connected, simply connected, complete, C^{Ω} Riemann manifolds of dimension $n \geqq 2$. Assume that the minimal differential polynomials of M, $\overline{M}$ coincide. Let $p \in M$, $\bar{p} \in \overline{M}$, and assume that p, $\bar{p}$ are not singular base points. If there exists a linear isometry $I: T_p(M) \to T_{\bar{p}}(\overline{M})$ which preserves the curvatures of M, $\overline{M}$ and their first $4n-5$ covariant differentials, then M and $\overline{M}$ are isometric by an isometry $h: M \to \overline{M}$ satisfying $h(p) = \bar{p}$, $(h_*)_p = I$.*

With this theorem we obtain a characterization of homogeneous Riemann manifolds:

THEOREM 3.4. *Let M be a connected, simply-connected, complete, C^{ω} Riemann manifolds of dimension $n \geqq 2$. Assume that M is infinitesimally homogeneous of order $3n-5$ in the sense that for any points p, q of M there exists a linear isometry $I: T_p(M) \to T_q(M)$ which preserves the curvature and its first $3n-5$ covariant differentials. Then M is homogeneous.*

This theorem improves a result of Singer [10], which states that if M is infinitesimally homogeneous of order $\dfrac{n(n-1)}{2}$, then M is homogeneous.

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