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TWO-NUMBER OF SYMMETRIC *R*-SPACES

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Dedicated to Professor Shingo Murakami on his sixtieth birthday

Introduction

Chen-Nagano [2] introduced a Riemannian geometric invariant $\nu(M)$, called the 2-*number*, for a compact (connected) symmetric space M: Points $p, q \in M$ are said to be *antipodal* to each other, if $p = q$ or there is a closed geodesic of M on which p and q are antipodal to each other. A subset A of M is called an *antipodal subset* if every pair of points of A are antipodal to each other. Now the 2-number $\nu(M)$ is defined as the maximum possible cardinality $|A|$ of an antipodal subset A of M. The 2-number is finite.

In this note we will prove the following

THEOREM. *If M is a symmetric R-space* (*See* § 1 *for the definition*), *we have*

$$\nu(M) = \dim H(M, Z_2)\,,$$

where $H(M, Z_2)$ denotes the homology group of M with coefficients $\boldsymbol{Z}_2$.

§ 1. Symmetric *R*-spaces

A compact symmetric space M is said to have a *cubic lattice* if a maximal torus of M is isometric to the quotient of $\boldsymbol{R}^r$ by a lattice of $\boldsymbol{R}^r$ generated by an orthogonal basis of the same length. A Riemannian product of several compact symmetric spaces with cubic lattices is called a *symmetric R-space*. We here recall some properties of symmetric R-spaces (cf. Takeuchi [4], [6], Loos [2]).

A symmetric R-space M has the complexification $\overline{M}$: There exists uniquely a connected complex projective algebraic manifold $\overline{M}$ defined over $\boldsymbol{R}$ such that the set $\overline{M}(\boldsymbol{R})$ of $\boldsymbol{R}$-rational points of $\overline{M}$ is identified

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