Z. J. Jurek
Nagoya Math. J.
Vol. 114 (1989), 53–64

RANDOM INTEGRAL REPRESENTATIONS FOR CLASSES OF LIMIT DISTRIBUTIONS SIMILAR TO LÉVY CLASS L_0, II

ZBIGNIEW J. JUREK[1]

§ 0. Introduction

Let $\xi(t)$ and $\eta(t)$ be two stochastic processes such that ξ has stationary independent increments and $\xi(0) = 0$ a.s. Suppose that $\xi(1) \stackrel{d}{=} t\xi(t^\beta) + \eta(t)$ for each $0 < t \leq 1$, with $\xi(t^\beta)$ independent of $\eta(t)$ and a fixed parameter $\beta \in (-2, 0)$. It is shown that $\xi(1)$ satisfies the above equation if and only if $\xi(1)$ is a sum of two independent r.v.'s: strictly stable one with the exponent $-\beta$ and the one given by a random integral $\int_{(0,1)} t dY(t^\beta)$, where Y has stationary independent increments and $\boldsymbol{E}\,[\|Y(1)\|^{-\beta}] < \infty$.

The aim of this paper is to find a random integral representation for some classes of limit distributions. Such representations give a very natural connection between theory of limit distributions and theory of stochastic processes. In some sense this note complements the subject, with a long history, of characterizations of stochastic processes by random integrals; cf. B.L.S. Praksa Rao (1983). On the other hand, this is a continuation of the study begun in Jurek (1988) but basically in a case of a Hilbert space and the identity operator. Recall that an infinitely divisible measure μ belongs to the class $\mathscr{U}_\beta$ if and only if

$$\forall (0<c<1) \exists \mu_c \in ID, \qquad \mu = T_c \mu^{*c^\beta} * \mu_c\,. \tag{0.1}$$

Here T_c is the linear operator of multiplying by a scalar c. In terms of stochastic processes the equation (0.1) can be rewritten as follows: There exist processes $\xi(t)$ and $\eta(t)$ such that ξ has stationary independent increments ($\mu = \mathscr{L}(\xi(1))$ and $\xi(1) \stackrel{d}{=} c\xi(c^\beta) + \eta(c)$ for $0 < c \leq 1$, with $\eta(c)$

Received August 15, 1986.
Revised February 8, 1988.
[1] This research was partially conducted at the Center for Stochastic Processes at the University of North Carolina, Chapel Hill N.C., USA, and supported by AFOSR Grant No. F49620 85 C 0144.