S. Kaneyuki and H. Asano
Nagoya Math. J.
Vol. 112 (1988), 81-115

GRADED LIE ALGEBRAS AND GENERALIZED JORDAN TRIPLE SYSTEMS

SOJI KANEYUKI AND HIROSHI ASANO

Dedicated to Professor Akihiko Morimoto on his sixtieth birthday

Introduction

One frequently encounters (real) semisimple graded Lie algebras in various branches of differential geometry (e.g. [16], [9], [14], [18]). It is therefore desirable to study semisimple graded Lie algebras, including those which have been studied individually, in a unified way. One of our concerns is to classify (finite-dimensional) semisimple graded Lie algebras in a way that enables us to construct them. A graded Lie algebra $\mathfrak{g}$ of the form $\mathfrak{g} = \sum_{k=-\nu}^{\nu} \mathfrak{g}_k$ is said to be of the ν-th kind. The classification of semisimple graded Lie algebras of the ν-th kind was done by Kobayashi-Nagano [4] for $\nu = 1$, and by J.H. Cheng [3] for $\nu = 2$ and $\dim \mathfrak{g}_{-2} = 1$. The first aim of this paper is to obtain a classification theorem (Theorem 1.7) for semisimple graded Lie algebras, which establishes a bijective correspondence between isomorphism classes of all gradations in a real semisimple Lie algebra $\mathfrak{g}$ and certain equivalence classes of partitions $(\Pi_0, \Pi_1, \cdots, \Pi_s)$ of a restricted fundamental root system Π of $\mathfrak{g}$. For the complex semisimple case, a similar but weaker assertion has been obtained by V.G. Kac [5]. Theorem 1.7 and its proof enable us to construct all gradations in a semisimple Lie algebra. A graded Lie algebra $\mathfrak{g} = \sum_{k=-\infty}^{\infty} \mathfrak{g}_k$ (not necessarily of finite dimension) is said to be of type α_0, if $\sum_{k \leqslant -1} \mathfrak{g}_k$ and $\sum_{k \geqslant 1} \mathfrak{g}_k$ are generated by $\mathfrak{g}_{-1}$ and $\mathfrak{g}_1$ respectively. In Theorem 2.6 we give a necessary and sufficient condition for a gradation to be of type α_0. By using this, we will construct explicitly (up to isomorphisms) all gradations of the first and the second kind in each classical real simple Lie algebra (§§2.3 and 4.2).

Our second concern is the problem of classifying a wider class of triple systems, called generalized Jordan triple systems which contain all

Received April 22, 1987.