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THE DIMENSION FORMULA OF THE SPACE OF CUSP FORMS OF WEIGHT ONE FOR $\Gamma_0(p)$

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Introduction

The purpose of this paper is to study the dimension formula for cusp forms of weight one, following the series of Hiramatsu [2] and Hiramatsu-Akiyama [3]. We define as usual the subgroup $\Gamma_0(N)$ of $SL_2(Z)$ by

$$\Gamma_0(N) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, Z) | c \equiv 0 \pmod N\right\}.$$

In this paper we consider the case of a prime level, so we always put $\Gamma = \Gamma_0(p)$ for a prime number p. In Section 1, we define the Eisenstein series and determine the constant term matrix explicitly. In Section 2, we calculate the trace of certain invariant integral operator by the method of Selberg. First we define a Selberg type zeta function $z(\delta, \chi)$ in (2.4). It appears in the trace formula from the hyperbolic conjugacy classes. After analytic continuation, we take the residue of $z(\delta, \chi)$ at 0 and get

$$\dim S_1(\Gamma, \chi) = \frac{1}{4} \operatorname*{Res}_{\delta=0} z(\delta, \chi).$$

On the other hand, Deligne-Serre (c.f. Serre [8]) proved that this dimension is equal to the number of two dimensional Galois representations satisfying certain conditions. Thus the residue of the Selberg type zeta function contains an information of the number of such representations.

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§ 1. Eisenstein series

Let k be a positive integer and χ a Dirichlet character modulo p. We

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