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ON THE MODULAR VERSION OF ITO'S THEOREM ON CHARACTER DEGREES FOR GROUPS OF ODD ORDER*

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§ 1. Introduction

One of the most useful theorems in classical representation theory is a result due to N. Ito, which can be stated using the classification of the finite simple groups in the following way.

THEOREM (N. Ito, G. Michler). *Let* $\mathrm{Irr}(G)$ *be the set of all irreducible complex characters of the finite group* G *and* q *be a prime number. Then* $q \nmid \chi(1)$ *for* $\chi \in \mathrm{Irr}(G)$ *if and only if* G *has a normal, abelian Sylow-q-subgroup.*

Ito himself proved the "if-part" in [7] and the "only-if-part" for p-solvable groups in [6]. To prove the last one in general, it is sufficient to investigate simple groups G (cf. Issacs [5] 12.33). For those, G. Michler [8] was able to prove that for $q \neq 2$ they all have q-blocks of non-maximal defect, which implies the result. For $q = 2$ he could show that each non-cyclic simple group has at least one character the degree of which is even.

Now replace the field C of complex numbers by any algebraically closed field K of characteristic $p > 0$ and denote by $\mathrm{IBr}(G)$ the Brauer characters of G with respect to p. The question which arises is, whether there is an analogue to the theorem above for $\mathrm{IBr}(G)$ instead of $\mathrm{Irr}(G)$. But now there are two different cases to consider, namely $q = p$ and $q \neq p$. The answer to the first one is quite satisfactory.

THEOREM. *We have* $p \nmid \beta(1)$ *for all* $\beta \in \mathrm{IBr}(G)$ *if and only if* G *has a normal Sylow-p-subgroup.*

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