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GENERALIZED RADON TRANSFORM AND LÉVY'S BROWNIAN MOTION, II*)

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§ 1. Introduction

As a continuation of the author's paper [19], we shall investigate the null spaces of a dual Radon transform R^*, in connection with a Lévy's Brownian motion X with parameter space (R^n, d). We shall follow the notation used in (I), [19].

We begin with a brief review of the general framework behind the representation of Chentsov type:

$$X(x) = \int_{B_x} W(dh) = W(B_x)\,, \tag{1}$$

with $B_x := \{h \in H;\ x \in h\}$. It consists of the following:

(i) A Lévy's Brownian motion $X = \{X(x);\ x \in M\}$ with mean 0 and variance $d(x, y) = E[(X(x) - X(y))^2]$, where $d(x, y)$ is an L^1-embeddable (semi-)metric on M;

(ii) A Gaussian random measure $W = \{W(dh);\ h \in H\}$ based on a measure space (H, ν) such that $H \subset 2^M$ and ν is a positive measure on H satisfying $\nu(B_x) < \infty$ and

$$d(x, y) = \nu(B_x \triangle B_y) = \int_H \pi_h(x, y)\nu(dh) \qquad \text{for all } x, y \in M\,, \tag{2}$$

where

$$\pi_h(x, y) := |\chi_h(x) - \chi_h(y)| = |\chi_{B_x}(h) - \chi_{B_y}(h)|\,.$$

As a bridge connecting the metric space (M, d) and the measure space (H, ν), the equation (2) guarantees the existence of a representation of the form (1) for a Lévy's Brownian motion X with parameter space (M, d).

The representation (1) of Chentsov type played in (I) (and will play

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