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A CHARACTERIZATION OF COMPLETE RIEMANNIAN MANIFOLDS MINIMALLY IMMERSED IN THE UNIT SPHERE*

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§1. Introduction

Let M^n be an n-dimensional Riemannian manifold minimally immersed in the unit sphere $S^{n+p}(1)$ of dimension $n+p$. When M^n is compact, Chern, do Carmo and Kobayashi [1] proved that if the square $\|h\|^2$ of length of the second fundamental form h in M^n is not more than $\frac{n}{2-1/p}$, then either M^n is totally geodesic, or M^n is the Veronese surface in $S^4(1)$ or M^n is the Clifford torus $S^k(\sqrt{k/n}) \times S^{n-k}(\sqrt{(n-k)/n})$ in $S^{n+1}(1)\,(0 < k < n)$.

In this paper, we generalize the results due to Chern, do Carmo and Kobayashi [1] to complete Riemannian manifolds.

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§2. Preliminaries

Let M^n be an n-dimensional Riemannian manifold which is minimally immersed in the unit sphere $S^{n+p}(1)$ of dimension $n+p$. Then the second fundamental form h of the immersion is given by $h(X, Y) = \tilde{\nabla}_X Y - \nabla_X Y$ and it satisfies $h(X, Y) = h(Y, X)$, where $\tilde{\nabla}$ and ∇ denote the covariant differentiations on $S^{n+p}(1)$ and M^n respectively, X and Y are vector fields on M^n. We choose a local field of orthonormal frames $e_1, \ldots, e_{n+p}$ in $S^{n+p}(1)$ such that, restricted to M^n, the vectors $e_1, \ldots, e_n$ are tangent to M^n. We use the following convention on the range of indices unless otherwised stated: $A, B, C, \cdots = 1,2,\ldots, n+p$; $i, j, k, \cdots = 1,2,3,\ldots,n$; $\alpha, \beta, \cdots = n+1, \ldots, n+p$. We agree the

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