J. Peetre
Nagoya Math. J.
Vol. 130 (1993), 183–192

HANKEL KERNELS OF HIGHER WEIGHT FOR THE BALL

JAAK PEETRE

The purpose of this note is to write down the general form of Hankel kernels for the complex unit ball $\mathbf{B}$ in $\mathbf{C}^d$. In the one dimensional case (unit disk $\varDelta$ in $\mathbf{C}$) this was done in [JP] and our treatment below has been guided by the insights gained there, and later, in a slightly different context, in [P]. We begin by summarizing the relevant facts in the case of the disk in a form convenient for us.

1. The disk revisited

Let $A^{\alpha,2}$ be the Hilbert space of analytic functions on $\varDelta$ which are square integrable with respect to the probability measure $d\mathcal{M}_a(z) = (\alpha + 1)(1 - |z|^2)^\alpha d\mathscr{E}(z)$ ($\mathscr{E}$ = Euclidean measure with an appropriate normalization, viz. $\int_\varDelta d\mathscr{E}(z) = 1$). In [JP] we considered bilinear forms on $A^{\alpha,2}(\varDelta)$ of the form

$$H(f_1, f_2) = \int\int_{\varDelta\times\varDelta} \overline{\mathscr{A}(z_1, z_2)} f_1(z_1) f_2(z_2)\, d\mathcal{M}_a(z_1)\, d\mathcal{M}_\alpha(z_2),$$

where the kernel $\mathscr{A}$ is given by

$$\mathscr{A}(z_1, z_2) = \frac{1}{2\pi i}\int_{\mathrm{T}} \frac{(z_1 - z_2)^s}{(w - z_1)^r (w - z_2)^r}\, b(w)\, dw,$$

where $r = \alpha + 2 + s$ ($s \in \mathbf{N}$). We write now this formula as

$$\mathscr{A}(z_1, z_2) = \frac{1}{2\pi i}\int_{\mathrm{T}} \frac{(z_1 - z_2)^s}{(1 - \bar{w}z_1)^r (1 - \bar{w}z_2)^r}\, b(w)\, \bar{w}^{2r} dw.$$

By partial integration one finds, quite generally, that

$$\int_\varDelta \bar{f} g\, d\mathcal{M}_{m-1} = \frac{(m+1)!}{2\pi i}\int_{\mathrm{T}} \bar{f} b \bar{w}^{m+1} dw$$

Received June 13. 1988.