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Nagoya Math. J.
Vol. 130 (1993), 85–100

# ASYMPTOTIC DEPENDENCE OF MOVING AVERAGE TYPE SELF-SIMILAR STABLE RANDOM FIELDS*

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## 1. Introduction and main results

As non-Gaussian stable stochastic processes have infinite second moments, one cannot use the covariance function to describe their dependence structure. We focus instead on the function

$$
\begin{aligned}
(1.1)\quad r(u) &= r(X, u; \theta_1, \theta_2) \\
&= E\exp\{i[\theta_1 X(u) + \theta_2 X(0)]\} \\
&\quad - E\exp\{i[\theta_1 X(u)\}\, E\exp\{i[\theta_2 X(u)\},\ u \in \mathbf{R}^1;\ \theta_1, \theta_2 \in \mathbf{R}^1,
\end{aligned}
$$

which is defined for any stationary process $\{X(u), u \in \mathbf{R}^1\}$.

This paper investigates the asymptotic behavior, as $u \to \infty$, of $r(u)$ for a large class of self-similar stable processes obtained as 'projections' of random fields. The function $r(u)$ is the difference between the characteristic function of the vector $(X(u), X(0))$ and the product of the characteristic functions of $X(u)$ and $X(0)$; it vanishes if and only if $X(u)$ and $X(0)$ are independent. If $\{X(u)\}$ is a Gaussian process, then $r(u)$ is asymptotically proportional to the covariance, provided the latter tends to zero, as $u \to \infty$. (See Levy and Taqqu [6], Theorem 1.1.)

The present section contains definitions, statements of the main results, and some comments. The proofs are given in Section 2.

A random field $\{X(t), t \in \mathbf{R}^n\}$ is called $S\alpha S$ (symmetric $\alpha$-stable) if any linear combination $\sum_{j=1}^{d} \theta_j X(t_j)$ has a symmetric stable distribution. We say that $\{X(t), t \in \mathbf{R}^n\}$ is self-similar with exponent $H$ if

$$
(1.2)\qquad \forall c > 0\quad \{X(ct), t \in \mathbf{R}^n\} \stackrel{d}{=} \{c^H X(t), t \in \mathbf{R}^n\},
$$

and has stationary increments if

Received May 6, 1992.

* The first author is on leave from the Hugo Steinhaus Center, Poland. The second author was partially supported by the ONR Grant N00014-90-J-1287 at Boston University and by a grant of the United States-Israel Binational Science Foundation.