

A NOTE ON THE VARIATION OF RIEMANN SURFACES

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1. Let X be any Riemann surface. By Koebe's uniformization theorem we know that the universal covering space of X is conformally equivalent to either Riemann sphere, complex plane, or the unit disc in the complex plane. If X is allowed to vary with parameters we may inquire the parameter dependence of the corresponding family of the universal covering spaces.

2. This question was essentially answered by the theory of simultaneous uniformization due to L. Bers [B]. According to this theory, for any complex analytic family of compact Riemann surfaces of genus ≥ 2 over a simply connected complex manifold M , the universal covering of the total space of the family is biholomorphically equivalent to a locally pseudoconvex domain in $M \times \mathbf{C}$ on which the covering transformation group acts as a complex analytic family of quasi-Fuchsian groups. This picture was extended by Earle and Fowler [E-F] to a certain class of variations of open Riemann surfaces.

3. On the other hand, H. Yamaguchi [Y] pursued the variation of open Riemann surfaces of class O_{AD} along the ideas initiated by K. Oka and T. Nishino, and succeeded in realizing the simultaneous Schottky uniformization of compact Riemann surfaces of genus ≥ 2 parametrized over the unit disc, completely independently of the method of Bers.

4. Therefore one may ask for a generalization of Yamaguchi's theory to open Riemann surfaces, or as an ultimate purpose for a simultaneous uniformization theory for an arbitrary Kleinian group.

5. Since few things seem to be known about such a generalized uniformization problem, we would like to proceed first by establishing a general result that holds for any covering space of a complex analytic family of compact Riemann