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ON THE INTERACTING FREE FOCK SPACE AND THE DEFORMED WIGNER LAW

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§1. Introduction

The Fock space is a basic structure for the quantum field theory and quantum stochastic calculus. In all the cases, a Fock space can be described as a direct sum of a sequence of some Hilbert spaces, i.e. a Fock space has the form of $\bigoplus_{n=0}^{\infty}\mathscr{H}_n$,where, $\mathscr{H}_0 := \mathbf{C}$ is the complex field and $\mathscr{H}_1 := \mathscr{H}$ is a given Hilbert space. Moreover,

i) the Boson Fock space corresponds to the case of $\mathscr{H}_n := \mathscr{H}^{\circ n}$, where $\mathscr{H}^{\circ n}$ is the n-folds symmetric tensor product of $\mathscr{H}$;

ii) the Fermion Fock space corresponds to the case of $\mathscr{H}_n := \mathscr{H}^{\wedge n}$, where $\mathscr{H}^{\wedge n}$ is the n-folds anti-symmetric tensor product of $\mathscr{H}$;

iii) the Free (or Full) Fock space corresponds to the case of $\mathscr{H}_n := \mathscr{H}^{\otimes n}$: the usual n-folds tensor product of $\mathscr{H}$.

In the consideration of the central limit of the time evolution operator of the quantum electric-magnetic field (see [1,2]), a new type of Free Fock structure is erquired in order to describe the limit of the time evolution operator. The limit is a quantum stochastic process satisfying a certain quantum stochastic differential equation.

In the new Free Fock structure, the n-th space $\mathscr{H}_n$ is not exactly equal to the n-folds tensor product Hibert space $\mathscr{H}^{\otimes n}$: one obtains the $\mathscr{H}_n$ by introducing a scalar product $\langle\cdot, \cdot\rangle_n$ on the algebraic tensor product $\mathscr{H}^{\odot n}$ and for $f_1, g_1, \ldots, f_n, g_n \in \mathscr{H}$, in general, the product $\Pi_{h=1}^{n}\langle f_k, g_k\rangle$ is not the same as the scalar product $\langle f_1 \odot \cdots \odot f_n, g_1 \odot \cdots \odot g_n\rangle$, where, we have omitted the sub-index n of n-th scalar product and throughout the paper, the same omission will be adopted.

Thus, the Free Fock (one could also make the same consideration for the Boson and the Fermion cases, see [3]) space has the form

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