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FINITE ARITHMETIC SUBGROUPS OF GL_n, V

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Abstract. Let K be a finite Galois extension of the rational number field $\mathbf{Q}$ and G a $Gal(K/\mathbf{Q})$-stable finite subgroup of $GL_n(O_K)$. We have shown that G is of A-type in several cases under some restrictions on K. In this paper, we show that it is true for $n = 2$ without any restrictions on K.

Let K be a finite Galois extension of the rational number field $\mathbf{Q}$ with Galois group Γ and let G be a Γ-stable finite subgroup of $GL_n(O_K)$. Here O_K stands for the ring of integers in K and we define the action of $\sigma \in \Gamma$ on $g = (g_{ij}) \in GL_n(O_K)$ by $\sigma(g) := (\sigma(g_{ij}))$. G being Γ-stable means that $\sigma(g) \in G$ for every $\sigma \in \Gamma$ and every $g \in G$. To state the property of such a group, we introduce the notion of A-type. Let H be a subgroup of $GL_n(O_K)$. We denote by $L = \mathbf{Z}[e_1, \ldots, e_n]$ a free module over $\mathbf{Z}$ and we make $h = (h_{ij}) \in H$ act on $O_K L$ by $h(e_i) = \sum_{j=1}^{n} h_{ij} e_j$. If there exists a decomposition $L = \oplus_{i=1}^{k} L_i$ such that for every $h \in H$, we can take roots of unity $\epsilon_i(h)$ $(1 \leq i \leq k)$ and a permutation $s(h)$ so that $\epsilon_i(h) h L_i = L_{s(h)(i)}$ for $i = 1, 2, \ldots, k$, then we say that H is of A-type.

We have shown in [4] that if Γ is nilpotent, then G is of A-type. The aim of this paper is to show the following

THEOREM. *Let K be a finite Galois extension of the rational number field $\mathbf{Q}$ with Galois group Γ and let G be a Γ-stable finite subgroup of $GL_2(O_K)$. Then G is of A-type.*

Through this paper, algebraic number fields are finite over the rational number field $\mathbf{Q}$. For an algebraic number field K, we denote the ring of integers in K by O_K. When K is the rational number field $\mathbf{Q}$, we use $\mathbf{Z}$ instead of $O_{\mathbf{Q}}$, as usual. An algebraic number field is called abelian if it is a Galois extension over $\mathbf{Q}$ with abelian Galois group. Let K be an algebraic number field and $\mathfrak{p}$ an integral ideal of K, and let G be a subgroup of $GL_n(O_K)$. Then we set

$$G(\mathfrak{p}) := \{g \in G \mid g \equiv 1_n \bmod \mathfrak{p}\},$$

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