ON MAXIMALLY CENTRAL ALGEBRAS

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Introduction

Let A be a primary algebra with unit element over a field K and Z its center. Let $\bar{A}$ be the simple residue class algebra of A modulo its radical. Then it is known, and can readily be seen, that there holds the inequality $[A : K] \geqq t^2[Z : K]$, where t is the rank of $\bar{A}$ over its center. We call A maximally central if in particular $[A : K] = t^2[Z : K]$ i.e. if the rank $[Z : K]$ takes its maximum value. Further, an algebra which is a direct sum of those primary algebras will be called maximally central too. The notion was introduced in Azumaya-Nakayama [5], as a by-product of the study of absolutely uni-serial algebras.

In the present paper, we shall investigate maximally central algebras as a main subject. For this purpose, it seems very natural to the writer to extend the definition of these from coefficient fields to coefficient rings.[1] From this view point, we consider throughout this paper algebras[2] over coefficient rings, and show that maximally central algebras behave quite similarly as simple algebras in the theory of ordinary algebras. In the former part of this paper, we introduce, after some considerations about general rings and algebras, the notion of maximally central algebras over general coefficient rings in an apparently different way from above, and in the latter part we confine ourselves to particular type of coefficient rings called Hensel rings. Our methods used in this paper are related not only to the algebraic theory of ordinary algebras but also to the arithmetical theory of $\mathfrak{p}$-adic algebras, particularly obtained by Witt and Nakayama.[3]

The main object of this paper is however to prove, in the last section 7, an existence theorem of inertial algebras, which may be seen as a generalization of the Wedderburn-Malcev's theorem[4] as well as that of Nakayama's theorem.[5]

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[1] Cf. also the footnote (3) in Azumaya-Nakayama [5].

[2] As for the term "algebra," see p. 125 below.

[3] Witt [13], Nakayama [12].

[4] Albert [1], III, Theorem 23; Deuring [5], II, § 11, Satz 1; Malcev [10].

[5] Nakayama [11], Satz 3.