ON THE FUNDAMENTAL EXISTENCE THEOREM OF KISHI

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1. Notation and terminology. Let Ω be a locally compact Hausdorff space and $G(x, y)$ be a strictly positive lower semicontinuous function on the product space $\Omega\times\Omega$ of Ω. Such a function $G(x, y)$ is called a ***kernel*** on Ω. The ***adjoint kernel*** $\check{G}(x, y)$ of $G(x, y)$ is defined by $\check{G}(x, y)=G(y, x)$. Whenever we say a measure on Ω, we mean a positive regular Borel measure on Ω. The ***potential*** $G_\mu(x)$ and the ***adjoint potential*** $\check{G}_\mu(x)$ of a measure μ relative to the kernel $G(x, y)$ is defined by

$$G_\mu(x)=\int G(x, y)\,d\mu(y) \quad\text{and}\quad \check{G}_\mu(x)=\int\check{G}(x, y)\,d\mu(y)$$

respectively. These are also strictly positive lower semicontinuous functions on Ω provided $\mu\neq 0$.

We say that a kernel $G(x, y)$ on Ω satisfies the ***continuity principle*** when, for any measure μ with compact support S_μ, the finite continuity of the restriction of $G_\mu(x)$ to S_μ implies the global finite continuity of $G_\mu(x)$ on Ω.

A property is said ***to hold G-p.p.p.*** on a subset X in Ω, when the property holds on X except a set E which does not contain any compact support S_ν of a measure $\nu\neq 0$ with finite G-energy $\int G_\nu(x)\,d\nu(x)$. Notice that $\int G_\nu(x)d\nu(x)=\int\check{G}_\nu(x)\,d\nu(x)$. Hence the notion G-p.p.p. is equivalent to that of $\check{G}$-p.p.p.

2. Result. M. Kishi [4] [5] proved the following important existence theorem in the potential theory with non-symmetric kernel:

Assume that the adjoint kernel $\check{G}(x, y)$ of $G(x, y)$ satisfies the continuity principle. Given a non-empty separable compact subset K of Ω and a strictly positive finite upper semicontinuous function $u(x)$ on K. Then there exists a measure μ with support S_μ in K such that

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