0. Introduction

Let \((M, g)\) be a Kähler \(C\)-space. \(R\) and \(\nabla\) denote the curvature tensor and the Levi-Civita connection of \((M, g)\), respectively.

In [6], Takagi have proved that there exists an integer \(n\) such that

\[ \tilde{\nabla}^{n-1} R \neq 0, \quad \tilde{\nabla}^n R \neq 0, \]

where \(\tilde{\nabla}\) denotes the covariant derivative of \((1,0)\)-type induced from \(\nabla\) (see Section 3 for the definition). Moreover, Takagi classified Kähler \(C\)-spaces with \(n = 2\) (Hermitian symmetric spaces of compact type are characterized as Kähler \(C\)-spaces with \(n = 1\)).

However, there is a mistake in deduction to lead a certain formula. The purpose of this paper is to correct the mistake and to classify Kähler \(C\)-spaces with \(n = 2\). Moreover, in Section 5, we shall classify Kähler \(C\)-spaces with \(n = 3\).

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1. Preliminaries

Let \(G\) be a Lie group and \(K\) a closed subgroup of \(G\). Let \(\mathfrak{g}\) and \(\mathfrak{k}\) be the Lie algebras of \(G\) and \(K\), respectively. Suppose that \(\text{Ad}(K)\) is compact. Then there exist an \(\text{Ad}(K)\)-invariant decomposition \(\mathfrak{g} = \mathfrak{t} + \mathfrak{p}\) of \(\mathfrak{g}\) and an \(\text{Ad}(K)\)-invariant scalar product \(\langle , \rangle\) on \(\mathfrak{p}\). Then

\[ [\mathfrak{t}, \mathfrak{p}] \subset \mathfrak{p} \]

\[ \langle [u, x], y \rangle + \langle [u, y], x \rangle = 0 \quad (u \in \mathfrak{t}, \; x, \; y \in \mathfrak{p}). \]

Moreover, under the canonical identification of \(\mathfrak{p}\) with the tangent space \(T_o(G/K)\) \((o = \{K\})\) of homogeneous space \(G/K\), the scalar product \(\langle , \rangle\) can be extended to