TRANSFORMATIONS ON WHITE NOISE FUNCTIONS ASSOCIATED WITH SECOND ORDER DIFFERENTIAL OPERATORS OF DIAGONAL TYPE

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Abstract. A generalized number operator and a generalized Gross Laplacian are introduced on the basis of white noise distribution theory. The equicontinuity is examined and associated one-parameter transformation groups are constructed. An infinite dimensional analogue of \( ax + b \) group and Cauchy problems on white noise space are discussed.

Introduction

During the recent development of infinite dimensional analysis much attention has been paid to roles of differential operators, in particular, of infinite dimensional Laplacians. Among many variants of Laplacians on an infinite dimensional space, the Gross Laplacian, the number operator (or the Beltrami Laplacian) and the Lévy Laplacian have been studied extensively. Based on an abstract Wiener space, Gross [9] introduced an infinite dimensional Laplacian (presently called the Gross Laplacian) and studied differential equations in infinite dimension. The number operator being a central object in quantum physics, the associated Cauchy problem was discussed by Piech [22], [23]. It is also noteworthy from harmonic analysis that the number operator appears as a limit of the spherical Laplacians as the dimension tends to the infinity [24], [25]. As for the Lévy Laplacian [17] a series of new interesting features has been investigated recently [1], it is still somehow beyond our discussion of this paper.

It was Kuo [13] who first made an attempt to understand these Laplacians in a unified manner, namely, as operators acting on white noise functions [10]. Later on it was proved that the Gross Laplacian \( \Delta_G \) and the number operator \( N \) are in essence the only operators which are rotation-