Nihonkai Math. J.
Vol.9 (1998), 27–46

ON EXISTENCE OF SOLUTIONS OF NONDEGENERATE WAVE EQUATIONS WITH NONLINEAR DAMPING TERMS

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Abstract In this paper, we consider the existence and asymptotic behavior of solutions of the following problems;

$$\begin{aligned}
&u_{tt}(t,x) - M(\|\nabla u(t,x)\|_2^2 + \|\nabla v(t,x)\|_2^2)\Delta u(t,x) + \delta|u_t(t,x)|^{p-1}u_t(t,x) \\
&\quad = \mu|u(t,x)|^{q-1}u(t,x), \quad x \in \Omega, \quad t \geq 0, \\
&v_{tt}(t,x) - M(\|\nabla u(t,x)\|_2^2 + \|\nabla v(t,x)\|_2^2)\Delta v(t,x) + \delta|v_t(t,x)|^{p-1}v_t(t,x) \\
&\quad = \mu|v(t,x)|^{q-1}v(t,x), \quad x \in \Omega, \quad t \geq 0, \\
&u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \Omega, \\
&v(0,x) = v_0(x), \quad v_t(0,x) = v_1(x), \quad x \in \Omega
\end{aligned}$$

where $q > 1, p \geq 1, \delta > 0, \mu \in R, \Delta$ the Laplacian in R^N, $M(s) = a + bs^\gamma, a + b \geq 0, b \geq 0$ and $\gamma \geq 1$.

Keywords and Phrases Existence and uniqueness, asymptotic behavior, degenerate wave equation, Galerkin method.

1. Introduction

Let Ω be a bounded domain in R^N with smooth boundary $\partial\Omega$. In this paper, we consider the existence of solutions of the following problems;

$$\begin{aligned}
&u_{tt}(t,x) - M(\|\nabla u(t,x)\|_2^2 + \|\nabla v(t,x)\|_2^2)\Delta u(t,x) + \delta|u_t(t,x)|^{p-1}u_t(t,x) \\
&\quad = \mu|u(t,x)|^{q-1}u(t,x), \quad x \in \Omega, \quad t \geq 0, \\
&v_{tt}(t,x) - M(\|\nabla u(t,x)\|_2^2 + \|\nabla v(t,x)\|_2^2)\Delta v(t,x) + \delta|v_t(t,x)|^{p-1}v_t(t,x) \\
&\quad = \mu|v(t,x)|^{q-1}v(t,x), \quad x \in \Omega, \quad t \geq 0, \\
&u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \Omega, \\
&v(0,x) = v_0(x), \quad v_t(0,x) = v_1(x), \quad x \in \Omega
\end{aligned} \tag{1.1}$$

where $q > 1, p \geq 1, \delta > 0, \mu \in R, \Delta$ the Laplacian in R^N, $M(s) = a + bs^\gamma, a + b \geq 0, b \geq 0$ and $\gamma \geq 1$.

1991 *Mathematics Subject Classification*. 35L70, 35L15, 65M60 .
This is supported by KOSEF 1996 .