

# ON A COMPOSITION OPERATOR AND HARDY SPACE

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**ABSTRACT.** Characterizing a geometric property of the self map that induces a bounded composition operator on Blochs to a Hardy-Sobolov space, we give a way of constructing examples of Bloch functions  $f$  whose derivative is in  $H^p$  for all  $p : 0 < p < 1$  but  $f \notin BMOA$ . The hyperbolic version of such an example is also given.

## 1. Introduction.

Let  $U = \{z : |z| < 1\}$  be the open unit disc of the complex plane and let  $T$  be the boundary of  $U$  identified with  $[-\pi, \pi]$ . Let  $\sigma(z)$  denotes the hyperbolic distance of  $z$  and 0 in  $U$ :

$$\sigma(z) = \frac{1}{2} \log \frac{1 + |z|}{1 - |z|}.$$

For  $0 < p \leq \infty$  and for  $f$  subharmonic in  $U$ , we set

$$\|f\|_p = \sup_{0 \leq r < 1} M_p(r, f),$$

where

$$M_p(r, f) = \left( \int_0^{2\pi} |f(re^{i\theta})|^p \frac{d\theta}{2\pi} \right)^{\frac{1}{p}} \quad \text{if } p < \infty$$

and

$$M_\infty(r, f) = \sup_{\theta} |f(re^{i\theta})|.$$

If  $f(z)$  is subharmonic in  $U$ , then it has a harmonic majorant if and only if  $\|f\|_1 < \infty$ .

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1991 *Mathematics Subject Classification*. Primary 30D55; secondary 28A25 and 30D50.

*Key words and phrases*. Bloch function, BMOA, Hardy space, composition operator.

This work was supported by NON DIRECTED RESEARCH FUND, Korea Research Foundation 1995 .

Typeset by  $\mathcal{A}\mathcal{M}\mathcal{S}$ -TEX