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Strong Convergence Theorems for Nonexpansive Nonself-mappings in Banach Spaces

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1 Introduction

Let E be a Banach space and let C be a nonempty closed convex subset of E. A mapping T from C into E is called nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. For a given $u \in C$ and each $t \in (0,1)$, we define a contraction $T_t : C \to E$ by

$$T_t x = tTx + (1-t)u \qquad \text{for all } x \in C. \tag{1}$$

If $T(C) \subset C$, then $T_t(C) \subset C$. Thus, by Banach's contraction principle , there exists a unique fixed point x_t of T_t in C, that is, we have

$$x_t = tTx_t + (1-t)u. \tag{2}$$

A question naturally arises to whether $\{x_t\}$ converges strongly as $t \to 1$ to a fixed point of T. This question has been investigated by several authors; see, for example, Browder[1], Halpern[4], Singh and Watson[8], Marino and Trombetta[6], and others. Recently, Xu and Yin[10] proved that if C is a nonempty closed convex subset of a Hilbert space H, if $T : C \to H$ is a nonexpansive nonself-mapping, and if $\{x_t\}$ is the sequence defined by (2) which is bounded, then $\{x_t\}$ converges strongly as $t \to 1$ to a fixed point of T. Next, consider a sunny nonexpansive retraction P from E onto C. Then, following Marino and Trombetta[6], for a given $u \in C$ and each $t \in (0,1)$, we define contractions S_t and U_t from