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ON SOME CR SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR FIELD IN A COMPLEX SPACE FORM

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Abstract We study CR submanifolds with nonvanishing parallel mean curvature vector field immersed in a complex space form.

Introduction One of typical submanifolds of a Kaehlerian manifold is the so-called *CR submanifolds* which are defined as follows: Let M be a submanifold of a Kaehlerian manifold $\tilde{M}$ with almost complex structure J. If there is a differentiable distribution such that it is invariant and the complementary orthogonal distribution is totally real (cf. [1], [2]). Especially, if each normal space of M is mapped into the tangent space under the action of J, M is called a *generic* submanifold of $\tilde{M}$. Real hypersurface of a Riemannian manifold are the most typical example of the generic submanifold ([13]).

Many subjects for CR submanifold were investigated from various different points of view. In [1, 2, 3, 4, 11] Bejancu, Chen, Kon and Yano studied basic properties of CR submanifolds M in a Kaehlerian manifold. In particular, under the assumptions that the second fundamental forms are commutative with the f-structure induced in the tangent bundle, some characterizations and some classifications of CR submanifolds with parallel mean curvature vector field in a complex space form were obtained (sec [7, 8, 9, 10]).

The purpose of the present paper is to study CR submanifolds of a complex space form with nonvanishing parallel mean curvature vector field under the assumption that the shape operator in the direction of the mean curvature vector field is commutative with the f-structure induced in the tangent bundle.

1. Preliminaries

Let $\tilde{M}$ be a Kaehlerian manifold of real dimension $2m$ equipped with an almost complex structure J and a Hermitian metric tensor G. Then for any vector fields X and Y on $\tilde{M}$, we have

$$J^2X = -X, \quad G(JX, JY) = G(X, Y), \quad \tilde{\nabla}J = 0,$$

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