Nihonkai Math. J.
Vol.6 (1995), 207-214

FORMALLY INTEGRABLE MIZOHATA SYSTEMS IN R^3

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In this article we investigate the local structure of Mizohata's systems in R^3, i.e. the local structure of systems of the form

$$\left| \begin{array}{l} \partial_1 u = \epsilon_1 i x^1 \partial_3 u + f \\ \partial_2 u = \epsilon_2 i x^2 \partial_3 u + g, \end{array} \right. \tag{1}$$

where $u(x) = u(x^1, x^2, x^3)$ is the unknown complex function of $x = (x^1, x^2, x^3) \in R^3$, $f(x)$ and $g(x)$ are given smooth complex functions, $\epsilon_j = +1$ or -1, $j = 1, 2$, and $\partial_k u = \frac{\partial u}{\partial x^k}$.

The local existence of solutions of Mizohata systems of codimension 1 in R^n was studied in the paper [1] by P. Cordaro and J. Hounie, where sufficient conditions for the local solvability were given in the case when the given functions in the right hand sides of the equations (f and g in (1)) satisfy certain conditions. The key problem for these systems, which are a special case of the complex linear systems of PDEs, is the problem of their solvability. As in the case of the classical Frobenius theorem, a necessary condition for the solvability of such systems is their *formal integrability*, i.e. the compatibility of their right hand sides; for the system (1) these conditions are

$$L_1 g = L_2 f, \tag{2}$$

where $L_1 = \partial_1 - \epsilon_1 i x^1 \partial_3$ and $L_2 = \partial_2 - \epsilon_2 i x^2 \partial_3$.

The purpose of this note is to show that any system (1), satisfying (2), can be reduced by a suitable change of the variables to a system (possibly non-linear) in R^2, i.e. to a system, involving only two independent variables and two unknown functions, which is not overdetermined. This result relates the local theory of the formally integrable Mizohata systems with the local theory of the first order systems of PDEs in R^2. More exactly, the theory of the systems of the form (1), having solutions, is a part of the theory of the systems of PDEs with two unknown real functions of two real variables.

In view of this result any formally integrable system of the form (1) is equivalent to an ordinary first order smooth complex equation of the form $\partial w = F(z, w, \frac{\partial w}{\partial \bar{z}})$, where w is the unknown complex function of the complex variable z.

Note that, given a system of partial differential equations, it is often possible to simplify its equations by increasing the number of the variables (independent and dependent). The converse, i.e. to simplify the equations with a simultaneous decreasing the number of the variables is rarely possible; in the present article we consider a class of systems of the form (1), for which the procedure suggested below leads to a simplification.

It should be noted, that the general idea of the method, which we use here, as well as the key notion of characteristic vector fields, have been introduced by S. Lie.