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ADJOINT FAMILIES IN TOPOLOGICAL VECTOR SPACES

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ABSTRACT. Given a linear partial differential operator L of order m with c^m-coefficients and a distribution T on an open set Ω of $\mathbb{R}^n$, a necessary and sufficient condition is derived for the existence of a function $f \in L^p(\Omega)$, $1 < p < \infty$, such that $Lf = T$ in the sense of distribution.

1. Introduction

Suppose B is a reflexive Banach space, E a locally convex space and $T : B \to E$ a linear map (continuous or not). We obtain a necessary and sufficient condition so that given $g \in E$, there exists $f \in B$ such that $Tf = g$.

This result is applied to the problem of finding a solution of $f \in L^p(\Omega)$, $1 < p < \infty$ and Ω open in $\mathbb{R}^n$, for the differential equation $Lf = T$ where L is a partial differential operator of order m with c^m-coefficients and T is a distribution defined on Ω.

2. A Preliminary result in a Hilbert space

Proposition 1. Let T be a bounded linear operator on a Hilbert space H. Then given $g \in H$, there exists an $f \in H$ such that $Tf = g$ if and only if $\sup_{\|u\|=1} \frac{|(g,u)|}{\|T^*u\|}$ is finite.

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