Nihonkai Math. J.
Vol.6 (1995), 115-128

# NUMERICAL SOLUTION OF BOUNDARY VALUE PROBLEMS FOR RETARDED DIFFERENTIAL EQUATIONS WITH A PARAMETER

Tadeusz Jankowski

One step methods combined with an iterative method are applied to find a numerical solution of boundary value problems for retarded ordinary differential equations with a parameter. This paper deals with the convergence of such methods. Some estimates of errors are given too.

**1. Introduction.** We consider the system of retarded ordinary differential equations

$$(1)\qquad y'(t) = f(t, y(t), y(\alpha_1(t)), \cdots, y(\alpha_r(t)), \lambda), \quad t \in J = [a, b], \quad a < b,$$

where $f : J \times R^{q(r+1)} \times R^p \to R^q$ and $\alpha_i : J \to R$ are continuous and $\alpha_i(t) < t$, $t \in J$, $i = 1, 2, \cdots, r$. Here $\lambda \in R^p$ is a parameter. We assume that the solution of (1) is given on $J_a$, so

$$(2)\qquad y(t) = \Psi(t), \quad t \in J_a = [\bar{a}, a], \quad \bar{a} = \inf_{t \in J}\{\alpha_i(t),\ i = 1, 2, \cdots, r\} \quad \Psi \in C^1(J_a, R^q).$$

Here $C^1(J_a, R^q)$ denotes the space of all functions of the class $C^1$ defined on $J_a$ with a range in $R^q$. We are interested in the solution of (1-2) that satisfies the nonlinear boundary condition

$$(3)\qquad g(\lambda, y(b)) = \Theta_p, \quad \Theta_p \text{ is zero element in } R^p,$$

where $g : R^p \times R^q \to R^p$. By a solution of (1-3) we mean a function $\varphi \in C^1(J, R^q)$ and a parameter $\lambda \in R^p$ such that (1-3) to be satisfied. Problem (1-3) may also be named as an eigenvalue problem for retarded differential equations or as a problem of terminal control. Sometimes $g$ may be linear with respect to its variables or may depend on $\lambda$ or $y(b)$ only.

The question of existence and uniqueness of solutions of problems with parameters is alredy investigated (see, for example, [3, 8, 9, 10]). Due to this fact it will be assumed that our problem has the exact solution $(\varphi, \lambda)$. A numerical approximation of this solution is a task of this paper.

Notice that $\varphi$ is a function of $\lambda$. It is known that if $f$ has continuous first order partial derivatives with respect to the last $r + 2$ variables, then

$$Y(t; \lambda) \equiv \frac{\partial}{\partial \lambda}\varphi(t; \lambda)$$

is the solution of the problem

$$(4)\qquad \begin{cases} Y'(t;\lambda) = f_0(t, \varphi(t), \varphi(\alpha_1(t)), \cdots, \varphi(\alpha_r(t)), \lambda)Y(t;\lambda) + \\ \qquad + \sum_{i=1}^{r} f_i(t, \varphi(t), \varphi(\alpha_1(t)), \cdots, \varphi(\alpha_r(t)), \lambda)Y(\alpha_i(t);\lambda) + \\ \qquad + f_\lambda(t, \varphi(t), \varphi(\alpha_1(t)), \cdots, \varphi(\alpha_r(t)), \lambda), \quad t \in J, \\ Y(a;\lambda) = 0_{q\times p}. \end{cases}$$