Nihonkai Math. J.
Vol.7 (1996), 185-195

ON REAL HYPERSURFACES IN QUATERNIONIC PROJECTIVE SPACE WITH $\mathcal{D}^{\perp}$-PARALLEL SECOND FUNDAMENTAL FORM

JUAN DE DIOS PÉREZ AND YOUNG JIN SUH*

ABSTRACT. In this paper we give a complete classification of real hypersurfaces in a quaternionic projective space QP^m satisfying certain conditions on the orthogonal distribution $\mathcal{D}$.

§1. Introduction

Throughout this paper let us denote by M a connected real hypersurface in a quaternionic projective space QP^m,$m\geq 3$, endowed with the metric g of constant quaternionic sectional curvature 4. Let N be a unit local normal vector field on M and $U_i = -J_iN$, $i = 1,2,3$, where $\{J_i\}_{i=1,2,3}$ is a local basis of the quaternionic structure of QP^m, [3]. Several examples of such real hypersurfaces are well known, see for instance ([1],[5],[6],[7]).

Now, let us consider the following conditions that the second fundamental tensor A of M in QP^m may satisfy

$$(\nabla_X A)Y = -\Sigma_{i=1}^3\{f_i(Y)\phi_iX + g(\phi_iX, Y)U_i\}, \tag{1.1}$$

$$g((A\phi_i - \phi_iA)X, Y) = 0, \tag{1.2}$$

for any $i = 1,2,3$, and any tangent vector fields X and Y of M, where the connection of M induced from the connection of QP^m is denoted by ∇.

Pak [7] investigated the above conditions and showed that they are equivalent to each other. Moreover he used the condition (1.1) to find a lower bound of $\|\nabla A\|$ for real hypersurfaces in QP^m. In fact, it was shown that $\|\nabla A\|^2\geq 24(m-1)$ for such hypersurfaces and the equality holds if and only if the condition (1.1) holds. In this case it was also known that M is locally congruent to a real hypersurface of type A_1 or A_2, which means a tube of radius r over QP^k $(1\leq k\leq m-1)$ in the notion of Berndt [1], and Martinez and the first author [5].

Now let us define a distribution $\mathcal{D}$ by $\mathcal{D}(x) = \{X\in T_xM : X\perp U_i(x), i = 1,2,3\}$,$x\in M$, of a real hypersurface M in QP^m, which is orthogonal to the structure vector fields $\{U_1, U_2, U_3\}$ and invariant with respect to structure tensors $\{\phi_1, \phi_2, \phi_3\}$, and by $\mathcal{D}^{\perp} = Span\{U_1, U_2, U_3\}$ its orthogonal complement in TM.

1991 *Mathematics Subject Classification.* Primary 53C40; Secondary 53C15.
* The second author was supported by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1995 and partly by TGRC-KOSEF.