Nihonkai Math. J.
Vol.7 (1996), 113-145

All solutions of the Diophantine equation $2^aX^r + 2^bY^s = 2^cZ^t$ where r, s and t are 2 or 4

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1. Introduction

We shall determine all solutions of the equation $2^aX^r+2^bY^s = 2^cZ^t$ in nonzero integers X, Y, Z, where a, b, c are non-negative integers, and r, s, t are 2 or 4, and X, Y, Z are pairwise relatively prime. To discuss the solutions of this equation, we may assume that X, Y, Z are all positive odd integers. We shall show that the following results.

The equations

$$\begin{array}{rcrcrl} X^2 & + & Y^2 & = & 2Z^2 & , \\ X^2 & + & 2^mY^2 & = & Z^2 & , \\ X^2 & + & Y^2 & = & 2Z^4 & , \\ X^2 & + & 2^mY^2 & = & Z^4 & , \\ X^2 & + & Y^4 & = & 2Z^2 & , \\ X^4 & + & 2^mY^2 & = & Z^2 & \end{array}$$

and

$$X^2 + 2^mY^4 = Z^2$$

have independently infinite solutions.

The equation

$$2^aX^4 + 2^bY^4 = 2^cZ^4$$

has only one trivial solution.

The equation

$$X^4 + Y^4 = 2Z^2$$

has only one trivial solution. (A.M.Legendre)

The equation

1991 *Mathematics Subject Classification.* Primary 11D41; Secondary 11D09.