CHARACTERIZATIONS OF REAL HYPERSURFACES IN COMPLEX SPACE FORMS IN TERMS OF WEINGARTEN MAP

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ABSTRACT. This paper consists of two parts. One is to give the notion of the ruled real hypersurfaces in a complex space form $M_n(c)$, $c \neq 0$ and to calculate the expression of the covariant derivative of its Weingarten map. Moreover, we know that real hypersurfaces of type A also satisfy this expression. The other is to show that ruled real hypersurfaces or real hypersurfaces of type A are the only real hypersurfaces in $M_n(c)$ which satisfy this expression.

1. Introduction

A complex *n*-dimensional Kaehlerian manifold of constant holomorphic sectional curvature *c* is called a complex space form, which is denoted by $M_n(c)$. A complete and simply connected complex space form consists of a complex projective space P_nC , a complex Euclidean space C^n or a complex hyperbolic space H_nC , according as c > 0, c = 0 or c < 0. The induced almost contact metric structure of a real hypersurface M of $M_n(c)$ is denoted by (ϕ, ξ, η, g) .

There exist many studies about real hypersurfaces of $M_n(c)$. One of the first researches is the classification of homogeneous real hypersurfaces of a complex projective space P_nC by Takagi [14], who showed that these hypersurfaces of P_nC could be divided into six types which are said to be of type A_1, A_2, B, C, D , and E, and in [3] Cecil-Ryan and [7] Kimura proved that they are realized as the tubes of constant radius over Kaehlerian submanifolds if the structure vector field ξ is principal. Also Berndt [2] showed recently that all real hypersurfaces with constant principal curvatures of a complex hyperbolic space H_nC are realized as the tubes of constant radius over certain submanifolds when the structure vector field ξ is principal. Nowadays in H_nC they are said to be of type A_0, A_1, A_2 , and B.

¹⁹⁹¹ Mathematics Subject Classification. Primary 53C40; Secondary 53C15.

This paper was supported by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1994 and partly by TGRC-KOSEF and BSRI-94-1404.