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A THIRD ORDER DIFFERENTIAL EQUATION AND REPRESENTABLE POLES

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Abstract It is showed in this note that if a third order differential equation $w''' = \lambda(z)w'' + R(z,w)w' = \lambda(z)w'' + \frac{P(z,w)}{Q(z,w)}w'$, where $\lambda(z)$ is a meromorphic function and $P(z,w)$ and $Q(z,w)$ are polynomials in w with meromorphic coefficients, possesses an admissible solution $w(z)$, then $w(z)$ satisfies a linear differential equation, a second order equation of Painlevé type, or first order equation of the form $c(z)(w')^2 + B(z,w)w' + A(z,w) = 0$, where $B(z,w)$ and $A(z,w)$ are polynomials in w having small coefficients with respect to $w(z)$. The main tools of the proof are lemmas on representable poles.

1. Introduction

In this note, we will treat algebraic differential equations with admissible solutions in the complex plane. The Malmquist–Yosida–Steinmetz type theorems have been studied by means of the Nevanlinna theory. During the last two decades several mathematicians gave remarkable improvements. We can find them, for instance, in Laine [8, Chapters 9–13].

In this note, we use standard notations in the Nevanlinna theory (see e.g. [2], [8], [10]). Let $f(z)$ be a meromorphic function. As usual, $m(r,f)$, $N(r,f)$, and $T(r,f)$ denote the proximity function, the counting function, and the characteristic function of $f(z)$, respectively.

A function $\varphi(r)$, $0 \leqq r < \infty$, is said to be $S(r,f)$ if there is a set $E \subset \mathbb{R}^+$ of finite linear measure such that $\varphi(r) = o(T(r,f))$ as $r \to \infty$ with $r \notin E$.

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