

On two-parameter discrete time optimal starting-stopping problems

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Abstract

We discuss the finiteness of an optimal stopping point and an ϵ -optimal stopping point for the discrete time two-parameter optimal stopping problem.

We also formulate the two-parameter optimal starting-stopping problem for the discrete time case. Further, several problems with the constraints on a starting time and a stopping time are studied, and several nested Bellman equations, named by Sun [11], are investigated.

Keywords : two-parameter stochastic process * starting-stopping problem * strategy * tactic * nested Bellman equation

1 Introduction

In this paper we study a stochastic control problem with not only a stopping time but also a starting time for two-parameter stochastic processes, which may be termed the two-parameter optimal starting-stopping problem.

The two-parameter optimal stopping problem has been studied by several authors.. Haggstrom [3], Krengel and Sucheston [4], Lawler and Vanderbei [5] and Mandelbaum and Vanderbei [7] formulated the optimal stopping problems for the stochastic processes indexed by a partially ordered set and solved them through the dynamic programming approach. That is to say, by introducing the Snell envelopes, which are well-known in the one-parameter optimal stopping problems, and the concept of a stopping point, a strategy and a tactic, they gave a construction of an optimal solution. Mandelbaum [6] formulated the optimal stopping problem for the multi-parameter stochastic processes and developed not only the well-known dynamic programming approach but also the theory of the dynamic allocation index.

In particular, Mandelbaum and Vanderbei [7] showed that if a partially ordered set is a two-dimensional non-negative lattice, the two-parameter optimal stopping