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On self-dual almost Hermitian 4-manifolds *

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1. Introduction

For an oriented Riemannian 4-manifold M, the space $\wedge^2 M$ of 2-forms on M splits with respect to the star operator $*$ into $\wedge^2 M = \wedge^2_+ M \oplus \wedge^2_- M$, where $\wedge^2_\pm M$ are eigenspaces corresponding to the eigenvalues ± 1. The Weyl conformal curvature tensor W viewed as an $End(TM)$-valued 2-form decomposes into $W = W_+ \oplus W_-$ and we say that M is *self-dual* if $W_- = 0$.

An almost Hermitian 4-manifold (M, g, J) is said to be *of pointwise constant holomorphic sectional curvature* if the holomorphic sectional curvature of M is constant for every unit tangent vectors and depends only on points of M. S. Tachibana ([7]) introduced the notion of Ricci $*$-tensor on an almost Hermitian manifold M, and we say that M is *weakly $*$-Einstein* if the Ricci $*$-tensor ρ^* takes the form $\rho^* = \lambda^* g$ for some differentiable function λ^* on M. In particular, if λ^* is constant on M, then M is said to be *$*$-Eisenstein* (see also [5] and [6]).

The main purpose of this paper is to prove the followings

Theorem A. *An almost Hermitian 4-manifold (M, g, J) is self-dual, and the components of the Ricci tensor ρ of M satisfy*

$$\rho_{11} + \rho_{22} = \rho_{33} + \rho_{44}, \rho_{14} = \rho_{23}, \rho_{13} + \rho_{24} = 0, \tag{1.1}$$

or the components of the Ricci $*$-tensor ρ^* of M satisfy

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