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ON A NEW CLASS OF UNIVALENT FUNCTIONS

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ABSTRACT

A well-known linear operator is defined which acts on an analytic function in the open unit disk by forming its convolution with an incomplete beta function. In this paper, using this operator, we define a new class of analytic functions in the unit disk and prove that this class consists entirely of univalent functions. An inclusion result is given. It is shown that it is closed under convolution with convex function and some applications of this result are also discussed.

1. INTRODUCTION

Let A be the class of analytic functions f on the open unit disk $E = \{z : |z| < 1\}$, normalized by $f(0) = 0$ and $f'(0) = 1$. The class A is closed under the Hadamard product or convolution

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^{n+1},$$

where

$$f(z) = \sum_{n=0}^{\infty} a_n z^{n+1}, \quad g(z) = \sum_{n=0}^{\infty} b_n z^{n+1}.$$

In particular, we consider convolution with the function $\phi(a,c)$ defined by

$$\phi(a,c;z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} z^{n+1}, \quad z \in E, \quad c \neq 0, -1, -2, \ldots.$$

where

$$(a)_n = \Gamma(a+n)/\Gamma(a),$$

i.e. $(a)_0 = 1$, $(a)_n = a(a+1)\ldots\ldots\ldots (a+n-1)$, $n \geq 1$. The function $\phi(a,c)$ is an incomplete beta function, related to the Gauss hyper-