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A Note on Actions of Compact Matrix Quantum Groups on von Neumann Algebras

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Abstract. In this paper we consider the object $\widetilde{S_\mu U(2)}$ coming from $S_\mu U(2)$ defined by S. L. Woronowicz, and construct an action of $\widetilde{S_\mu U(2)}$ on the Powers factor R_λ if $\lambda = \mu^2$. Moreover we show that the fixed point algebra under the action is the AFD II_1-factor which is generated by Jones projections.

1. Introduction

In [8] Woronowicz introduced a concept of a compact matrix quantum group (a compact matrix pseudogroup) which is a certain deformation of the dual object of compact groups. Let $G = (A, u)$ be a compact matrix quantum group and $\Phi : A \longrightarrow A \otimes_{min} A$ be a $*$-homomorphism called a comultiplication where A is a unital C^*-algebra as in [8]. The comultiplication Φ is an action of G on itself.

In [3] the author, Nagisa and Watatani constructed an action of G on the Cuntz algebra $\mathcal{O}_n$ or the UHF-algebra M_n^∞ of type n^∞. The forms of the actions ψ and ψ' were represented as follows :

$$\psi : \mathcal{O}_n \longrightarrow \mathcal{O}_n \otimes_{min} A, \qquad \psi' : M_n^\infty \longrightarrow M_n^\infty \otimes_{min} A.$$

Especially in [3] they considered the actions of $S_\mu U(2)$ (Woronowicz, [9]) on $\mathcal{O}_2$ and M_2^∞, and showed the fixed point algebras under the actions were generated by Jones projections. This means a C^*-algebra version of a deformation of the result of the case for the action of $SU(2)$ by Jones in [1] and [2].

In this paper we construct an action of $\widetilde{S_\mu U(2)}$ coming from $S_\mu U(2)$ on the Powers factor R_λ if $\lambda = \mu^2$ using the Kac-Takesaki operator introduced by Nakagami and Takesaki in [4] and [6]. Moreover we show that the fixed point algebra under the action is the AFD II_1-factor which is generated by the Jones projections $\{e_n\}_{n=1}^\infty$ such that

$$e_i e_{i\pm1} e_i = (\lambda + \lambda^{-1} + 2)^{-1} e_i,$$

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