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On p-quasihyponormal operators

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Abstract

For a p-quasihyponormal operator T with the polar decomposition $T = U|T|$, we show that $T_p = U|T|^p$ is quasihyponormal with spectrum $\sigma(T_p) = \{r^p e^{i\theta} : e^{i\theta} \in \sigma(T)\}$. From this, we obtain the following Putnam type inequality for a p-quasihyponormal operator T

$$\||T|^{2p} - |T^*|^{2p}\| \leq 2\|T\|^p \left(\frac{p}{\pi} \iint_{re^{i\theta} \in \sigma(T)} r^{2p-1}\, dr d\theta\right)^{\frac{1}{2}}.$$

These results are parallel with Xia, Aluthge and Chō-Itoh's results for p-hyponormal operators. Also we show that the Riesz idempotent E for T with respect to an isolated point λ of the spectrum $\sigma(T)$ satisfies $\mathrm{ran}E = \ker(T - \lambda)$, moreover, if $\lambda \neq 0$ then E is self-adjoint and $\ker(T - \lambda) = \ker(T - \lambda)^*$.

1. Introductions

Studying p-hyponormal operators, i.e., operators T on a (separable) complex Hilbert space $\mathcal{H}$ such that $(T^*T)^p \geq (TT^*)^p$, for $0 < p < 1$ was first started by D. Xia [20], in that paper, he gave an example of semi-hyponormal operator but not hyponormal. Here we say that an operator T is hyponormal iff T is 1-hyponormal, semi-hyponormal iff T

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