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WANDERING VECTORS OF FINITE SUBDIAGONAL ALGEBRAS

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ABSTRACT. In this note we consider wandering vectors and their multipliers for finite subdiagonal algebras. We prove that the set of completely wandering vectors of a finite subdiagonal algebra is connected and is closed if and only if the finite subdiagonal algebra is antisymmetric. We also prove that the set of all wandering vector multipliers for an antisymmetric finite subdiagonal algebra forms a group.

1. INTRODUCTION

The notion of wandering vectors was introduced in [1] by Arveson to study factorization in finite subdiagonal algebras. This notion is very useful in the study of analytic operator algebras (cf. [1, 3, 6, 10, 11, 12] and so on). On the other hand, wandering vectors and their multipliers for unitary systems are systemically studied by several authors (cf. [2, 4, 5, 7]). It is noted that the structure of wandering vectors of both subdiagonal algebras and unitary systems is very interesting. In this note we consider wandering vectors of a finite subdiagonal algebra.

Arveson introduced the notion of subdiagonal algebras to study the analyticity in operator algebras in [1]. Let $\mathcal{M}$ be a σ-finite von Neumann algebra on $\mathcal{H}$ and $\mathfrak{D}$ a von Neumann subalgebra of $\mathcal{M}$. Let Φ be a faithful normal conditional expectation from $\mathcal{M}$ onto $\mathfrak{D}$. A subalgebra $\mathfrak{A}$ of $\mathcal{M}$, containing $\mathfrak{D}$, is called a subdiagonal algebra of $\mathcal{M}$ with respect to Φ if

(i) $\mathfrak{A} \cap \mathfrak{A}^* = \mathfrak{D}$,
(ii) Φ is multiplicative on $\mathfrak{A}$, and
(iii) $\mathfrak{A} + \mathfrak{A}^*$ is σ-weakly dense in $\mathcal{M}$.

The algebra $\mathfrak{D}$ is called the diagonal of $\mathfrak{A}$. Although subdiagonal algebras are not assumed to be σ-weakly closed in [1], the σ-weak closure of a subdiagonal algebra is again a subdiagonal algebra of $\mathcal{M}$ with respect to Φ (Remark 2.1.2 in [1]). Thus we assume that our subdiagonal algebras are always σ-weakly closed. We say that $\mathfrak{A}$ is a

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