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From number systems to shift radix systems

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Abstract

Shift radix systems provide a unified notion to study two important types of number systems. In this paper, we briefly review the origin of this notion.

1. Introduction

Let $\mathbf{r} = (r_1, \ldots, r_d) \in \mathbb{R}^d$. Consider a mapping $\tau_{\mathbf{r}} : \mathbb{Z}^d \to \mathbb{Z}^d$, which maps each element $(z_1, \ldots, z_d)$ to $(z_2, \ldots, z_{d+1})$, provided that

$$0 \leq r_1 z_1 + r_2 z_2 + \cdots + r_d z_d + z_{d+1} < 1.$$

Obviously, $\tau_{\mathbf{r}}$ is defined by

$$\tau_{\mathbf{r}}((z_1, \ldots, z_d)) = (z_2, \ldots, z_d, -\lfloor r_1 z_1 + \cdots + r_d z_d \rfloor). \tag{1.1}$$

We say that $\tau_{\mathbf{r}}$ has the *finiteness property* if for every $\mathbf{z} \in \mathbb{Z}^d$ there exists a k, such that $\tau_{\mathbf{r}}^k(\mathbf{z}) = \mathbf{0}$.

This concept unifies notions for two important number systems, namely *canonical number systems* and *β-expansions.* For these number systems, the finiteness property means that all numbers of a certain set admit finite expansions. This property also plays an important role for constructing tilings giving the Markoff partitions of dynamical systems associated to these number systems.

If $\tau_{\mathbf{r}}$ has the finiteness property, then $(\mathbb{Z}, \tau_{\mathbf{r}})$ is called a *shift radix system (for short SRS)* (cf. [2, 3]). It turned out to be a hard problem to characterize all

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