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On Some EP Operators

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Abstract

Let H be Hilbert space, and let $T : H \to H$ be a bounded linear operator with closed range. In this paper, we introduce a new family of operators with generalized inverse $T^\dagger$ such that $T^\dagger T \geq TT^\dagger$, which is weaker than the case of EP. Moreover we characterize such operators and give some fundamental properties.

1 Introduction and preliminaries

Throughout this paper we assume that H_1, H_2, and H are separable complex Hilbert spaces with inner product $(\cdot,\cdot)$. Let $B(H_1, H_2)$ be the set of all bounded linear operators from H_1 into H_2. Let $B_C(H_1, H_2)$ be the subspace of all $T \in B(H_1, H_2)$ such that the range of T is closed in H_2. If $H_1 = H_2 = H$, we write $B(H) = B(H, H)$ and $B_C(H) = B_C(H, H)$. For $T \in B(H_1, H_2)$, $\ker T$ and $R(T)$ denote the kernel and the range of T, respectively.

According to Nashed [6], $T \in B_C(H_1, H_2)$ has a Moore-Penrose inverse $T^\dagger$, that is, $T^\dagger$ is the unique solution for the equations:

$$TT^\dagger T = T,\ T^\dagger TT^\dagger = T^\dagger,\ (TT^\dagger)^* = TT^\dagger,\ \text{and}\ (T^\dagger T)^* = T^\dagger T, \tag{1.1}$$

where T^* denotes the adjoint operator of T. Later of this, we write M-P inverse for short.

We need the following results of $T^\dagger$ and $R(T)$. See [3, 4, 5] for details.

Theorem A. (i) For any $T \in B_C(H_1, H_2)$ with M-P inverse $T^\dagger$, we have that

$$T^\dagger T = P_{R(T^\dagger)},\ TT^\dagger = P_{R(T)},\ (T^\dagger)^\dagger = T,\ \text{and}\ (T^\dagger)^* = (T^*)^\dagger,$$

where P_M is the orthogonal projection from H onto M.
(ii) For any $T \in B(H)$,
(1) $R(T)$ is closed if and only if $T^\dagger$ is bounded;
(2) $R(T)$ is closed if and only if $R(T^*)$ is closed.

An operator T in $B(H)$ is said to be an *EP operator* if the range of T is equal to the range of its adjoint T^*, i.e., $R(T) = R(T^*)$. For $S, T \in B(H)$, we write