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POISSON SUMMATION FORMULA FOR THE SPACE OF FUNCTIONALS

TAKASHI NITTA AND TOMOKO OKADA

Abstract

In the preceding work, we formulated a Fourier transformation on the infinite-dimensional space of functionals. Here we first calculate the Fourier transformation of infinite-dimensional Gaussian distribution $\exp\left(-\pi\xi\int_{-\infty}^{\infty}\alpha^2(t)dt\right)$ for $\xi \in \mathbf{C}$ with $\mathrm{Re}(\xi) > 0$, $\alpha \in L^2(\mathbf{R})$, using our formulated path integral. Secondly we develop the Poisson summation formula for the space of functionals, and define a functional Z_s, $s \in \mathbf{C}$, so that our path integral of the functional Z_s corresponds to Riemann's zeta function in the case that $\mathrm{Re}(s) > 1$.

0. Introduction

In the preceding paper([N-O2]), we defined a delta functional δ and a Fourier transformation F on the space of functionals in the infinitesimal analysis as one of generalizations for Kinoshita's infinitesimal Fourier transformation in the space of functions. Historically, in 1962, Gaishi Takeuchi([T]) introduced a δ-function for the space of functions under nonstandard analysis. In 1988, 1990, Kinoshita([K1],[K2]) defined his Fourier transformation in the infinitesimal analysis for the space of functions. He called it "an infinitesimal Fourier transformation". Nitta and Okada([N-O1],[N-O2]) defined, for funtionals, an infinitesimal Fourier transformation, using a concept of double infinitesimals, and calculated the infinitesimal Fourier transform for two typical examples. The main idea is to use the concept of double infinitesimals and taking standard parts twice st(st(.)). In our theory, the infinitesimal Fourier transform of δ, δ^2, ... , and $\sqrt{\delta}$, ... can be calculated as constant functionals, 1, infinite, ... , and infinitesimal, ...

Now let H be an even infinite number in ${}^*\mathbf{R}$, and L be a lattice with infinitesimal spacing

$L := \left\{\varepsilon z \,\middle|\, z \in {}^*\mathbf{Z},\ -\frac{H}{2} \le \varepsilon z < \frac{H}{2}\right\}$, where $\varepsilon = \frac{1}{H}$, and let H' be an even infinite number in ${}^*({}^*\mathbf{R})$, and L' be a lattice with infinitesimal spacing

$L' := \left\{\varepsilon' z' \,\middle|\, z' \in {}^*({}^*\mathbf{Z}),\ -\frac{H'}{2} \le \varepsilon' z' < \frac{H'}{2}\right\}$, where $\varepsilon' = \frac{1}{H'}$. We hereafter call a lattice with infinitesimal spacing, for short, *an infinitesimal lattice.*

Then we calculate the Fourier transform of a nonstandard functional of Gaussian type. The functional of Gaussian type means that the standard part of the image for $\alpha \in L^2$ is $\exp\left(-\pi\xi\int_{-\infty}^{\infty}\alpha^2(t)dt\right)$, for $\xi \in \mathbf{C}$ with $\mathrm{Re}(\xi) > 0$. We choose such a nonstandard functional and calculate the Fourier transform of it. Then the standard part of the Fourier transform satisfies that the image of $\alpha \in L^2$ is $C_\xi \exp\left(-\pi\xi^{-1}\int_{-\infty}^{\infty}\alpha^2(t)dt\right)$, in which C_ξ is a constant independent of b.