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Representation of Ammann-Beenker tilings by an automaton

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ABSTRACT. The Ammann-Beenker tilings are quasiperiodic tilings of the plane, which is constructed by using the Ammann's matching rules. We show that the Ammann-Beenker tilings can be composed by an automaton with 4 states, and note some results concerning composition sequences from the viewpoint of symbolic dynamics.

1. Introduction

In 1984 quasi-crystals with icosahedral symmetry were discovered by Shechtman et al.([9]). Before that, it had been believed that the structure of a crystal was periodic, like a wallpaper pattern. Periodicty is another name for translational symmetry. Icosahedral symmetry is incompatible with translational symmetry and therefore quasi-crystals are not periodic. The most famous 2-dimensional mathematical model for a quasi-crystal would be Penrose tilings of the plane ([6],[7]). The tiles of the Penrose's tilings are two types of rhombs with double and single arrows on the edges, as shown in Figure 1.

In the Penrose maching rules, the common edges of two adjacent tiles must have the same type (single or double) and the same direction of the arrows. The up-down generation introduced by J. Conway is one of the methods to construct such tilings or quasiperiodic tilings. In [3], de Bruijn actually uses the up-down generation to construct Penrose tilings, and represents the method by an automaton.

In this note we interested in Ammann-Beenker tilings (see [1],[2],[5] for example). Decorated Ammann-Beenker tiles are two rhombs with acute angles 45° and a square, and lengths of their edges are the same. Ammann-Beenker