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On totally real submanifolds of a complex projective space*

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Abstract

Montiel, Ros and Urbano [3] showed a complete characterization of compact totally real minimal submanifold M of $CP^n(c)$ with Ricci curvature S of M satisfying $S \geq \frac{3(n-2)}{16}c$. The purpose of this paper is to answer Ogiue's conjecture which the above result remains true under the weaker condition of the scalar curvature ρ of M satisfying $\rho \geq \frac{3n(n-2)}{16}c$.

1 Introduction.

Let $CP^n(c)$ be an n-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature c(> 0) and let M be an n-dimensional compact totally real minimal submanifold isometrically immersed in $CP^n(c)$. Let h be the second fundamental form of M in $CP^n(c)$.
Recently, Montiel, Ros and Urbano [3] proved the following: Let M be an n-dimensional compact totally real minimal submanifold isometrically immersed in $CP^n(c)$. Then the Ricci curvature S of M satisfies

$$S \geq \frac{3(n-2)}{16}c$$

if and only if one of the following conditions holds: a) $S = \frac{n-1}{4}c$ and M is totally geodesic, b) $S = 0, n = 2$ and M is a finite Riemannian covering of a flat torus minimally embedded in $CP^2(c)$ with parallel second fundamental form, c) $S = \frac{3(n-2)}{16}c$, $n > 2$ and M is an embedded submanifold congruent to the standard embedding of: $SU(3)/SO(3), n = 5; SU(6)/Sp(3), n = 14; SU(3), n = 8$; or $E_6/F_4, n = 26$.
Ogiue [5] conjectured the following: Under the weaker assumption of $\rho \geq \frac{3n(n-2)}{16}c$, the above result remains true, where ρ is the scalar curvature of M.
With respect to this conjecture the author [4] showed: Let M be an n-dimensional

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