Nihonkai Math. J.
Vol.13(2002), 133-143

Asymptotic Behavior of Solutions for a Delay Reaction-Diffusion Equation of Neutral Type*

B Shi

Abstract: In this paper, we consider a delay reaction-diffusion equation of neutral type of the form:

$$\frac{\partial}{\partial t}\left(u(t,x)+pu(t-\tau,x)\right)+q(t,x)u(t-\sigma,x)=a^2\Delta u(t,x) \qquad (*)$$

for $(t,x)\in\mathbb{R}^+\times\Omega$ with homogeneous Neumann boundary condition:

$$\frac{\partial}{\partial \mathbf{n}}u(t,x)=0 \text{ for } (t,x)\in\mathbb{R}^+\times\partial\Omega \qquad (**)$$

and initial condition:

$$u(t,x)=\phi(t,x) \text{ for } (t,x)\in[-\lambda,0]\times\bar{\Omega}, \qquad (***)$$

where $\tau>0,\sigma\in\mathbb{R}^+,p,a\in\mathbb{R},\lambda=\max\{\tau,\sigma\},q(t,x)>0$ for $(t,x)\in\mathbb{R}^+\times\Omega,\Omega$ is a bounded open region in $\mathbb{R}^n,\partial\Omega$ is the boundary of Ω, which is piecewise smooth, $\mathbf{n}$ is the exterior normal direction to $\partial\Omega$ and Δ is the Laplacian operator. We study various cases of p in the neutral term and obtain that if $p>1$ then every nonoscillatory solution of Initial and Boundary Value Problem (*)-(***) tends uniformly in $x\in\Omega$ to zero as $t\to\infty$; if $p=-1$ then every solution of Initial and Boundary Value Problem (*)-(***) oscillates and if $p<-1$ then every nonoscillatory solution of Initial and Boundary Value Problem (*)-(***) goes uniformly in $x\in\Omega$ to infinity or minus infinity under some hypotheses.

Keywords: asymptoic behavior, reaction-diffusion equations, delays, neutral type, oscillation

Mathematics Subject Classification (2000): 35B05, 35B40

1 Introduction

Consider the delay reaction-diffusion equation of neutral type of the form:

$$\frac{\partial}{\partial t}\left(u(t,x)+pu(t-\tau,x)\right)+q(t,x)u(t-\sigma,x)=a^2\Delta u(t,x) \qquad (1)$$

*supported by the Distinguished Expert Science Foundation of Naval Aeronautical Engineering Institute.