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ON THE RATE OF CONVERGENCE OF A POSITIVE APPROXIMATION PROCESS

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ABSTRACT. In this paper we are dealing with a class of summation integral operators on unbounded interval generated by a sequence $(L_n)_{n\geq 1}$ of linear and positive operators. We study the degree of approximation in terms of the moduli of smoothness of first and second order. Also we present the relationship between the local smoothness of functions and the local approximation. By using probabilistic methods, new features of $L_n f$ are pointed out such as the approximation property at discontinuity points and the monotonicity property under some additional assumptions of the function f. Also the rate of convergence of these operators for functions of bounded variation is given.

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1. Introduction

In [7] Lupaş proposed to study the following sequence of linear and positive operators

$$(L_n f)(x) = 2^{-nx} \sum_{k=0}^{\infty} \frac{(nx)_k}{2^k k!} f\left(\frac{k}{n}\right), \quad x \geq 0, \quad f : [0,\infty) \to \mathbf{R}, \tag{1}$$

where $(\alpha)_0 = 1$ and $(\alpha)_k = \alpha(\alpha+1)\dots(\alpha+k-1)$, $k \geq 1$.

We can consider that L_n, $n \geq 1$, are defined on E where $E = \bigcup_{a>0} E_a$ and E_a is the subspace of all real valued continuous functions f on $[0,\infty)$ such as $e(f;a) := \sup_{x\geq 0}(\exp(-ax)|f(x)|)$ is finite. The space E_a is endowed with the norm $\|f\|_a = e(f;a)$ with respect to which it becomes a Banach lattice.

Concerning the raised problem, in [1] some quatitative estimates for the rate of convergence were given. Among the results below we mention the following.