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AN APPLICATION OF FURUTA INEQUALITY

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In this note, by the Furuta inequality, we give an elementary proof of a result of Aluthge and Wang as follows: If T is an invertible w-hyponormal operator, then T^2 is also w-hyponormal.

1. Introduction. Through this note, let T be a bounded linear operator on a Hilbert space $\mathcal{H}$. For positive operators A and B, we write $A \geq B$ if $A - B \geq 0$. We denote $A \succ B$ if A and B are invertible positive operators satisfying $A \geq (A^{\frac{1}{2}}BA^{\frac{1}{2}})^{\frac{1}{2}}$. Let $T = U|T|$ be the polar decomposition of T. We define $\widetilde{T} = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$. The operator $\widetilde{T}$ is called the Aluthge transformation of T([1]). We denote $\widehat{T} = |\widetilde{T}|^{\frac{1}{2}}\widetilde{U}|\widetilde{T}|^{\frac{1}{2}}$, where $\widetilde{T} = \widetilde{U}|\widetilde{T}|$ is the polar decomposition of $\widetilde{T}$. An operator T is called w-hyponormal if $|\widetilde{T}| \geq |T| \geq |\widetilde{T}^*|$. The notion of w-hyponormal operators was introduced by Aluthge and Wang ([3],[5]). It is known that if T is p-hyponormal , then T^2 is not in general([2]). Indeed, Halmos gave an example of a hyponormal operator T for which T^2 is not hyponormal. But the situation is different for log-hyponormal operators : if T is log-hyponormal, then T^2 is log-hyponormal([4]). Since log-hyponormal operators are invertible w-hyponormal operators, it is natural to ask whether similar result holds for inverible w-hyponormal operators. Aluthge and Wang gave an affirmative answer to this question and proved the following result:

PROPOSITION A ([5], Theorem 5.2). *If T is an invertible w-hyponormal operator, T^2 is also w-hyponormal.*

The aim of this note is to give an elementary proof of Proposition A by the Furuta inequality ([6]).

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