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Real hypersurfaces in complex projective space satisfying a certain condition on Ricci tensors

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1 Introduction

Let $CP^n, n \geq 3$ be an n-dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 4, and let M be a real hypersurface CP^n. Let ν be a unit local normal vector field on M and $\xi = -J\nu$, where J denotes the complex structure of CP^n. M has an almost contact metric structure (ϕ, ξ, η, g) induced from J. We denote R and S the curvature tensor and the Ricci tensor of M, respectively. Many differential geometeres have studied M (cf. [1], [5], [6], [8], [9], [10], [11] and [12]) by using the structure (ϕ, ξ, η, g).

Typical examples of real hypersurfaces in CP^n are homogeneous ones. Takagi [12] showed that all homogeneous real hypersurfaces in CP^n are realized as the tubes of constant radius over compact Hermitian symmetric spaces of rank 1 or rank 2. Namely, he showed the following: Let M be a homogeneous real hypersurface of CP^n. Then M is a tube of radius r over one of the following Kaehler submanifolds:
(A_1) hyperplane CP^{n-1}, where $0 < r < \frac{\pi}{2}$,
(A_2) totally geodesic $CP^k (1 \leq k \leq n-2)$,
(B) complex quadric Q_{n-1}, where $0 < r < \frac{\pi}{4}$,
(C) $CP^1 \times CP^{\frac{n-1}{2}}$, where $0 < r < \frac{\pi}{4}$ and $n (\geq 5)$ is odd,
(D) complex Grassmann $CG_{2,5}$, where $0 < r < \frac{\pi}{4}$ and $n = 9$,
(E) Hermitian symmetric space $SO(10)/U(5)$, where $0 < r < \frac{\pi}{4}$ and $n = 15$.
Due to his classification, we find that the number of distinct constant principal curvatures of a homogeneous real hypersurface is 2, 3 or 5. Here note that the vector ξ of any homogeneous real hypersurface M (which is a tube of radius r) is a principal curvature vector with principal curvature $\alpha = 2\cot 2r$ with multiplicity 1 (See [1]) and that in the case of type A_1 M has two distinct principal curvatures and in the case of type A_2 (resp. B) M has three distinct principal curvatures $t, -\frac{1}{t}$ and $\alpha = t - \frac{1}{t}$ (resp. $\frac{1+t}{1-t}, \frac{t-1}{t+1}$ and $\alpha = t - \frac{1}{t}$).

The following result is well known ([3]) : There are no real hypersurfaces M with parallel Ricci tensor in $CP^n, n \geq 3$. Moreover, $CP^n, n \geq 3$, does not