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Regeneration in Quaternionic Analysis

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In Complex Analysis of Several Variables, Matsugu [6] gave a necessaryand sufficient condition that any pluriharmonic function g on a Rieman domain Ω over a Stein manifold is a real part of a holomorphic function on Ω. In Quaternionic Analysis, Nôno [8] gave a necessary and sufficient condition that any harmonic function f_1 on a domain Ω in C^2 has a hyper-conjugate harmonic function f_2 so that the function $f_1 + f_2 j$ is hyperholomorphic on Ω. Marinov [5] developped systematically a theory of regenerations of regular functions. The main purpose of the present paper is to add a regeneration in Quaternionic Analysis.

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1. Regeneration

Let Ω be a complex manifold and f be a holomorphic function on Ω. Then its real part f_1 is a pluriharmonic function on Ω. Let (Ω, φ) be a Rieman domain over a Stein manifold S and $(\tilde{\Omega}, \tilde{\varphi})$ be its envelope of holomorphy over S. Then, Matsugu [6] proved that the necessary and sufficient condition that, for any pluriharmonic function f_1 on Ω, there exists a pluriharmonic function f_2 on Ω so that $f_1 + f_2 i$ is holomorphic on Ω is that there holds $\mathrm{H}^1(\tilde{\Omega}, \mathrm{Z}) = 0$, where Z is the ring of integers.

The field $\mathcal{H}$ of quaternions

$$z = x_1 + ix_2 + jx_3 + kx_4, \qquad x_1, x_2, x_3, x_4 \in \mathrm{R} \tag{1}$$

is a four dimensional non-commutative R-field generated by four base elements 1, i, j and k with the following non commutative multiplication rule:

$$i^2 = j^2 = k^2 = -1, ij = -ji = k, jk = -kj = i, ki = -ik = j. \tag{2}$$

x_1, x_2, x_3 and x_4 are called, respectively, the real, i, j and k part of z. In the papers Nôno [7], [8], [9], [10] and Marinov [5] loco citato, two complex numbers

$$z_1 := x_1 + ix_2, \quad z_2 := x_3 + ix_4 \in \mathrm{C} \tag{3}$$

are associated to (1), regarded as

$$z = z_1 + z_2 j \in \mathcal{H}. \tag{4}$$

The quaternionic conjugate z^* of $z = z_1 + z_2 j \in \mathcal{H}$ is defined by

$$z^* := \overline{z_1} - z_2 j. \tag{5}$$