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NONLINEAR NONLOCAL TRANSPORT-DIFFUSION EQUATIONS ARISING IN PHYSIOLOGY

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ABSTRACT. We study a transport-diffusion initial value problem arising in mathematical models of muscle contraction. The equation has the transport term whose coefficient is a time function depending on the solution in a nonlinear and nonlocal way. In this paper, we investigate the unique existence of a strong solution in a function space BUC. Our results allow the inhomogeneous term to depend on the solution in a nonlinear way, such as $\gamma(t)f(x)(1-u^p)-g(x)u^q$ and $\gamma(t)f(x)(1-u)^p-g(x)u^q$.

Key words and phrases: Muscle contraction, nonlocal transport-diffusion equation, semilinear evolution equation

1. INTRODUCTION

In this paper we study the initial value problem for a nonlinear nonlocal transport-diffusion equation with a small diffusion coefficient $\varepsilon \in]0,1]$:

$$u_t - \varepsilon u_{xx} + z'(t)u_x = \varphi(x,t,z(t),u), \quad (x,t) \in \mathbb{R}\times[0,T], \tag{1.1}$$

$$z(t) = L\Big(\int_{\mathbb{R}} w(x)u(x,t)dx\Big), \quad t \in [0,T], \tag{1.2}$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}. \tag{1.3}$$

Here $u : \mathbb{R}\times[0,T] \to \mathbb{R}$ and $z : [0,T] \to \mathbb{R}$ are unknown functions, z' stands for the time-derivative. The functions φ, L, w and u_0 are given functions specified later.

Study of the above equation is related to the nonlinear nonlocal first order hyperbolic problem: Find $u : \mathbb{R}\times[0,T] \to \mathbb{R}$ and $z : [0,T] \to \mathbb{R}$ for which

$$u_t + z'(t)u_x = \varphi(x,t,z(t),u), \quad (x,t) \in \mathbb{R}\times[0,T] \tag{1.4}$$

and (1.2)–(1.3) are satisfied. This hyperbolic problem is formulated as a rheological model describing the so-called cross-bridge dynamics observed in the muscle contraction phenomena in physiology. For the model problem, see [1, 4, 5, 7, 8] and the references therein. The constitutive unit of muscle structure is called a sarcomere which consists of particles of myosin (thick filament) and actin (thin filament). The cross-bridges are chemical links between myosin and actin filaments. According to the sliding filament theory of Huxley [8], the phenomenon of muscle contraction is a consequence of relative sliding motion between these two filaments and this sliding occurs when the cross-bridges attach the actin filaments