Parallel submanifolds of Cayley plane

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1. Introduction

A submanifold M of a Riemannian manifold $\widetilde{M}$ is called to be parallel if the second fundamental form of M is parallel. Several authors have completely classified parallel submanifolds when the ambient spaces are the Euclidean space and the symmetric spaces of rank one except Cayley plane and its non-compact dual. Parallel submanifolds of the Euclidean space and the sphere have been classified by D. Ferus [2], [3], [4] and those of the real hyperbolic space by M. Takeuchi [12]. Parallel Kaehler submanifolds of the complex projective space and the complex hyperbolic space have been classified by H. Nakagawa and R. Takagi [11] and by M. Kon [6] respectively. H. Naitoh in [8], [9], and [10] has classified totally real parallel submanifolds of the complex space form and consequently has completely classified parallel submanifolds of the complex space form. Parallel submanifolds of the quaternion projective space and its non-compact dual have been classified by the author [13]. In this paper we will study parallel submanifolds of Cayley plane $P_2(\mathbf{Cay})$.

We need the classification of totally geodesic submanifolds of $P_2(\mathbf{Cay})$ to classify parallel submanifolds of $P_2(\mathbf{Cay})$. On this, the following result is obtained:

THEOREM (J. A. WOLF [14]). *Let N be a connected complete totally geodesic submanifold of Cayley plane $P_2(\boldsymbol{Cay})$ with dim $N \geqq 2$. Then N is an r-dimensional sphere S^r ($2 \leqq r \leqq 8$), a real projective plane $P_2(\boldsymbol{R})$, a complex projective plane $P_2(\boldsymbol{C})$, or a quaternion projective plane $P_2(\boldsymbol{H})$. Moreover, if two connected complete totally geodesic submanifolds are homeomorphic, then they are equivalent under an element of $I_0(P_2(\boldsymbol{Cay}))$, where $I_0(P_2(\boldsymbol{Cay}))$ denotes the identity component of the full group of isometries of $P_2(\boldsymbol{Cay})$.*

Especially maximal totally geodesic submanifolds of $P_2(\mathbf{Cay})$ are $P_2(\boldsymbol{H})$ and S^8.

In this paper we will show the following:

THEOREM. *Let f be an immersion with parallel second fundamental form of a connected manifold M (dim $M \geqq 2$) into Cayley plane $P_2(\boldsymbol{Cay})$. Then there exists a totally geodesic submanifold $P_2(\boldsymbol{H})$ or S^8 of $P_2(\boldsymbol{Cay})$ which contains the image $f(M)$ of M by f.*

By this Theorem a parallel submanifold M of $P_2(\mathbf{Cay})$ is reduced to either of the following cases: