On a modified Robbins-Monro procedure approximating the root from below with errors in setting the x-levels

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1. Introduction and Summary

In the case of finding the unique root θ of the equation $M(x)=0$, situations may occure where even the precise setting of the x-levels of an experiment is impossible without error. DUPAČ and KRÁL [3] and WATANABE [6] dealt with these situations. On the other hand, there are cases in which it is advantageous to use a process which converges to θ from below. ANBAR [1] gave a modified Robbins-Monro (RM) procedure for guaranteeing that with probability one the procedure overestimates θ only finitely many times. In this paper, it is shown that assuming that the error in x-level can be made small at some rate at each step, the modified RM procedure overestimates θ only finitely many times with probability one.

This paper consists of five sections. In section 2, we shall give some assumptions, notations and a lemma. In section 3, we shall show a convergence theorem. Section 4 will give some lemmas which are used in section 5. In section 5, we shall present two theorems which show that with probability one the modified RM process overestimates θ only finitely many times and give an asymptotic normality of the process.

2. Preliminaries

Let R be the real line. Let $\{U^n(x)\}$ and $\{V^n(x)\}$ be two sequences of random variables which depend on parameter $x\in R$. Suppose that for each n, $U^n(x)$ and $V^n(x)$ are measurable functions of x. Further, suppose $E[U^n(x)]=E[V^n(x)]=0$ for all $x\in R$ and all $n\geqslant 1$.

Let $M(x)$ be a real-valued measurable function on R, let θ be the unique root of $M(x)=\alpha$ where α is an arbitrary given number.

Let us define the mdified RM procedure proposed by ANBAR [1] as follows: Let X_1 be a random variable with $E[X_1^2]<\infty$ and let define $X_2, X_3, \cdots$ by the recursive relation

$$(2.\ 1)\qquad X_{n+1}=X_n-a_n[M(X_n+u_n)-v_n-\alpha+b_n]\quad n=1, 2, \cdots\cdots$$