Simply connected 6-manifolds of large degree of symmetry

By

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Introduction

For a compact connected differentiable manifold M, we define the degree of symmetry, denoted by $N(M)$, the maximum of dimensions of compact connected Lie groups which can act on M almost effectively.

In this note we shall determine simply connected 6-manifolds up to diffeomorphism whose degree of symmetry is greater than 5. Let M be a simply connected closed 6-manifold and G a compact connected Lie group acting almost effectively on M with dim $G=N(M)$. We may assume without loss of generality that G is a product $T^r\times G_1\times\cdots\times G_s$, where T^r is r-dimensional torus and G_i's are simply connected simple Lie groups. In [6], it is shown that if dim $G\geqq 12$, then G is transitive on M. In section 2, we shall determine simply connected 6-dimensional homogeneous spaces. Assume $N(M)\leqq 11$. Then we may consider only $Spin(5)$ $SU(3)$ and $SU(2)$ among the G_i's. It is shown that $r\leqq 5$. In section 3, we shall consider $Spin(5)$-actions, $SU(3)$-actions in section 4, $SU(2)\times SU(2)$-actions and $SU(2)\times SU(2)\times SU(2)$-actions in section 5 and $G\times T^r$-actions in section 6. We shall list the classification of simply conneted 6-manifolds by degree of symmetry in the last section.

Our initial aim was to find an exotic homotopy complex projective 3-space of large degree of symmetry. Our results show the following

Let M be a homotopy complex projective 3-*space. If $N(M)$ is greater than* 5, *then M is diffeomorphic to the standard complex projective* 3-*space.*

We can not determine degrees of symmetry of exotic homotopy complex projective 3-spaces.

1. Preliminaries

In this section we state some lemmas which are used in the sequel. Let (G, M) be a topological action. We denote by G_x the isotropy subgroup of G at $x\in M$, by $G(x)$ the orbit of x, $M^*=M/G$ the orbit space, by $F(G, M)$ the fixed point set and by $M_{(H)}$ the set of points x of M whose isotropy subgroup is conjugate to H.