

# ON A SAITO'S PROBLEM FOR THE GENERATIONS OF VON NEUMANN ALGEBRAS BY POWER PARTIAL ISOMETRIES

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## Abstract

Saito raised a problem whether any properly infinite von Neumann algebra acting on a separable Hilbert space is generated by a power partial isometry. We give the complete answer for this problem in this paper.

In this paper, we discuss the generation of von Neumann algebras by the power partial isometries. These were discussed by many authors, for example, Behncke, Wogen and Saito. In particular, Saito showed the following result in [4].

Let  $M$  be a properly infinite von Neumann algebra. Then, for any positive integer  $n$ , there exists a generator  $V$  of  $M$  satisfying the following properties;

- (1)  $V, V^2, \dots, V^n$  are non-zero partial isometries and  $V^k (k \geq n + 1)$  are not.
- (2)  $V$  is a nilpotent operator of index  $n + 3$ .

Furthermore, when Saito showed the above result, he raised the following problem in [4; Question in page 489]:

Let  $M$  be a properly infinite von Neumann algebra. Then, can we choose a generator  $T$  of  $M$  such that  $T, T^2, \dots, T^n, \dots$  are all non-zero partial isometries ?

For the above mentioned problem by Saito, in this paper we shall give the complete answer that the von Neumann algebra generated by the power partial isometry is the von Neumann algebra of type I and is not the von Neumann algebra of type  $II_\infty$  and type III.