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Local uniqueness of the unknown diffusivity in some heat conduction problem

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1. Introduction

Let us consider some heat conduction problem in a infinite composite material in $\mathbf{R}^3$. Suppose that the region $z > 0$ is of one material and $z < 0$ of another, and that there is no contact resistance at the boundary $z = 0$. Assume that a line source (which is identical with the y-axis) is emitting at the rate Q per unit length per unit time from time 0 to T. Measuring the temperature at the boundary, we want to determine the physical properties of one material when the physical properties of the other material are known. The uniqueness of the diffusivity of a finite homogeneous conductor in $\mathbf{R}^1$ was proved in [3]. In this note we shall give mathematical rigorous justification of the local uniqueness studied experimentally and theoretically in [1], [2], [6]. Our mathematical model is as follows:

$$(\partial_t - \kappa_1\Delta)u_1 = \frac{Q}{c_1\rho_1}\delta(x)\delta(z)\{H(t)-H(t-T)\}, -\infty < x, y < +\infty, z > 0, t > 0, T > 0, \quad (1.1)$$

$$(\partial_t - \kappa_2\Delta)u_2 = 0, \qquad -\infty < x, y < +\infty, z < 0, t > 0, \quad (1.2)$$

$$u_1(x, y, z, 0) = 0, u_2(x, y, z, 0) = 0, \quad (1.3)$$

$$u_1(x, y, 0, t) = u_2(x, y, 0, t), \quad (1.4)$$

$$\rho_1 c_1 \kappa_1 \partial_z u_1(x, y, 0, t) = \rho_2 c_2 \kappa_2 \partial_z u_2(x, y, 0, t), \quad (1.5)$$

where κ_1 (a positive constant) is the known diffusivity, ρ_1 and ρ_2 (positive constants) are the known density, c_1 and c_2 (positive constants) are the known specific heat, κ_2 (a positive constant) is the unknown diffusivity, and $H(t)$ is the Heaviside function. We measure the temperature at the boundary $z = 0$ excepting the y-axis (a line source).

$$\theta(x_0, y_0, T) \equiv u_2(x_0, y_0, 0, T), \; x_0 \neq 0. \quad (1.6)$$

Knowing $d\theta/d\log T$, we study the determination of κ_2. Our main theorem is as follows: